Optimal Allocation across Finite Alternatives: An application to International Portfolio Choice

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Abstract

Using large data on international equity portfolio allocations of international mutual funds, I observe a substantial fraction of reported zeroes. I discuss how to distinguish corner solutions from misreported zeroes in any optimal allocation problem across finite alternatives. While corner solutions matter for the analysis, misreported zeroes do not. Using the mandate behind a mutual fund's investment universe, I can identify corner solutions in my data. I motivate corner solutions with a N-asset model of optimal portfolio choice with short-selling constraints and heterogeneous investors. This model relates the portfolio share to expected future excess returns. I also extend the model with portfolio friction to account for persistence in portfolio choice. Then, I estimate the portfolio equations with fractional and dynamic fractional estimators. My results indicate that linear estimators produce large bias. OLS overestimate the persistence of past shares and underestimate the coefficient on expected excess returns. Linking the coefficients to the deep structural parameters of the model, OLS over-estimate both risk aversion and the parameter of portfolio frictions.

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1 Introduction

Working with data on optimal portfolio choice, I observe a substantial fraction of zeroes. Those zeroes might represent corner solutions which matter for any analysis or misreported zeroes which do not. For instance, when filling a report on her/his allocation, an individual (or more generally an entity) might not report any share for an alternative she/he does not consider. When aggregating all individuals' allocations, this non-reported share might be recoded as a zero. The objectives of this paper are to (i) identify the corner solutions from misreported zeroes and (ii) estimate their econometric importance in an application to international portfolio choice.

I start by showing we need two ingredients to identify corner solutions: (i) the individuals' alternative universe and (ii) an underlying model behind the optimal shares. In my discussion, reported zeroes arise for two reasons. In the first case, an individual reports zero shares for alternatives which are not in her/his alternative universe. In the second case, the individual reports zero shares for alternatives in the alternative universe whose optimal shares given by an underlying model hit the non-negative boundary conditions. To address the potential simultaneity between a model which solves the optimal shares in a N alternatives framework and the optimal N itself, I discuss we need to find applications in which N is given. Then, I apply this discussion to an application to international portfolio choice. Specifically, I study how mutual funds allocate their assets under management across countries. Because the investment universe of a mutual is given by a mandate [Koijen and Yogo, 2019], I can therefore take the investment universe as given and identify corner solutions in mutual funds data. I justify the corner solutions with a frictionless N-asset Markowitz mean-variance model with short-selling constraints. I also extend this model by introducing portfolio friction in the form of a costly deviation from past shares. This portfolio friction represents portfolio inertia. The model features three level of heterogeneity given by structural parameters. Investors differ in their risk aversion, portfolio risk and value of portfolio friction. Those two models give two simple and tractable portfolio equations. Remarkably, the coefficients of the models can be linked to the underlying structural parameters. The frictionless portfolio expression relates the country share to the average shares and expected future excess returns. The portfolio expression with friction relates the portfolio share to average shares, past shares and expected future excess returns. Funds with low risk aversion, low portfolio risk or low portfolio friction are more likely to report corner solutions. Those values are associated with a high coefficient on expected excess returns and a low coefficient on past shares.

I predict expected excess returns using macro and bond variables which are exogenous to portfolio shares. When predicting excess returns with financial variables, endogeneity problem might arise because of persistent financial shocks. For instance, a financial shock in a country create a global shift in portfolios. This global shift put a pressure on asset price which amplify the negative shock. By predicting excess returns with macro and bond variables, I ensure exogeneity.

I document the occurrence of corner solutions using a dataset on 1084 international equity mutual funds investing in 44 countries over the period 2002:01-2016:07. Corner solutions represent 25% of total observations (fund-month-country). However, at the fund level, the fraction of corner solutions varies substantially. While some funds report a lot of corner solutions, 8% of the funds always invest strictly positive shares. I split the funds by their characteristics. Active, small and broad funds report substantially more corner solutions than others. Bacchetta, Tièche, and van Wincoop [2020] shows active, small and broad funds are more responsive to expected excess returns and have a lower average country share (as they invest on average in more countries).

Given the structure of the data and the two different portfolio equations, I estimate the portfolio regressions with two different econometric models. I estimate the frictionless portfolio with the Bernoulli quasi maximum likelihood of Papke and Wooldridge [1996] and Papke and Wooldridge [2008]. I estimate the dynamic portfolio equation, the portfolio with frictions, with the two-limit random effect Tobit of Loudermilk [2007]. I contribute to the econometric literature by applying those estimators to portfolio choice. My results indicate that not taking into account the corner solutions lead to a large bias in estimating the coefficients. Even though I have large data, the bias introduced by misspecifiying the model is still important.

When relating the regression coefficients to the structural parameters of the model, I find that linear estimators over-estimate both risk aversion and the parameter of portfolio frictions. This is because linear estimator overestimate the persistence of portfolio shares and hence underestimate the coefficient on expected excess returns.

Finally, I contribute in the growing literature estimating international portfolio choice. Recently, Bacchetta, Tièche, and van Wincoop [2020] propose a model with portfolio frictions in which the optimal portfolio allocation depends on two benchmark portfolios and expected future economic conditions. Koijen and Yogo [2020] aggregate bilateral portfolio shares and regress those on a variety of financial and macroeconomic variables. Also using equity mutual funds, Raddatz and Schmukler [2012] regress portfolio allocations on some benchmark portfolios and valuation effects. However, their approach lack a clear theoretical framework. Camanho et al. [2018] also study the impact of valuation effects and past portfolio shares using another source of mutual funds data. This literature also relates to a strand of the literature estimating the link between international capital flows (as opposed to portfolio allocations) and past returns (e.g. Froot et al. [2001], Didier and Lowenkron [2012]). Outside the open economy literature, Bilias et al. [2010], Brunnermeier and Nagel [2008], Mitchell et al. [2006]), among others, use data on portfolio allocation by individual households. In the same spirit, Giglio et al. [2019] use survey data. In this micro literature, past portfolio is a key determinant of portfolio shares.

The rest of the paper starts with the identification of corner solutions.

2 The Identification of Corner Solutions

Imagine you have in your possession a large data on how much of a resource people share between several known finite alternatives. Specifically, each individual $i = 1, \ldots, I$ allocates a resource at date t across alternatives n in her or his alternative universe $N_{i,t} \in \{1, \ldots, N\}$. This alternative universe is a subset of all alternatives.

The individual chooses the optimal shares each period to maximize her or his objective function given non-negative constraints. Let $z_{i,n,t}$ represent the optimal share for n given by the model with $z_{i,n,t} \in [0, 1]$ and $\sum_{n=1}^{N_{i,t}} z_{i,n,t} = 1$.

$$z_{i,n,t} \in [0,1] \qquad \text{if } n \in N_{i,t} \tag{1}$$

$$= 0 \qquad \text{if } n \notin N_{i,t} \tag{2}$$

This model emphasizes that an individual reports a share of zero for two possible reasons. The first reason is that the individual does not consider the alternative because it is not in her or his alternative universe, equation (2). The second reason is that this is not optimal to allocate across an alternative even though the individual does consider it, equation (1). Therefore, corner solutions are the ones from equation (1).

As a result, we only need the individual alternative universe $N_{i,t}$ to identify corner solutions from misreported zeroes in the data. To tackle the simultaneity between the model which solves the optimal shares in a $N_{i,t}$ alternatives framework and the optimal $N_{i,t}$, we need to find applications in which $N_{i,t}$ is given. This is the case for international portfolio choice of mutual funds.

An Application to International Portfolio Choice

Mutual funds invest according to a mandate. This mandate represents the investment universe of the fund. Mutual funds also report the benchmark index they replicate. Using the benchmark index, I can identify the relevant investment universe of each fund. Let N_i^B represent the countries in the benchmark portfolio, N_i^+ represent the additional countries in which the fund invests in and N_i^- represent the countries in the benchmark index the fund does not invest in at all. The investment universe is given by equation (3)

$$N_i = N_i^B + N_i^+ - N_i^- \tag{3}$$

N_i^+ and N_i^- arise as some funds do not exactly invest in the same countries as their benchmark index.

For instance, a fund targets the MSCI Emerging Markets Latin America Index while another fund targets the MSCI Emerging Markets Europe 10/40 Index. Because MSCI reports the countries covered in each index, I can trace out the countries in which the fund is interested in.

2.1 Data

Using detailed panel data on international equity mutual funds, I find 25% of country shares of zero. Those equity funds report non-negative country allocations, portfolio returns[1] and assets under management to EPFR (Emerging Portfolio Fund Research) for 135 countries.[2]

Because I am concerned with introducing misreported zeroes as corner solutions, I drop, at the fund level, those countries in which the fund invests on average less than 0.01%. This drop countries in which the funds invests only for a few months. I also drop those countries in which less than 10 funds invest on average. This brings the focus on 44 developed and developing countries.[3] These countries represent on average more than 90% of international equity portfolios of those international mutual funds. The sample covers the period from January 2002 to July 2016.

In order to avoid backfill bias (see, Elton and Gruber [2013]), I drop those funds which have less than 5 millions USD in assets under management at the end of the sample and are reporting at most for 12 months. I keep funds domiciled in the US, the UK, France, Germany and in Ireland. This results in a sample of 1084 funds. I only look at the investments made in foreign countries to avoid the problem of home equity bias which is large in equity mutual funds data.

2.2 The Occurrence of Corner Solutions

Corner solutions represent 25.4% of the observations. This fraction is, however, heterogeneous across the funds. Figure 1a shows the distribution of country shares $z_{i,n,t}$. Corner solutions and small shares - those between 0 and 2% represent half of the data. An important feature of this data is the log-normal distribution of country shares. Using another data on portfolio choice, Koijen and Yogo [2019] also find log-linearity in the portfolio shares. This feature is key to finding the appropriate estimator in a portfolio regression. Next, I measure the

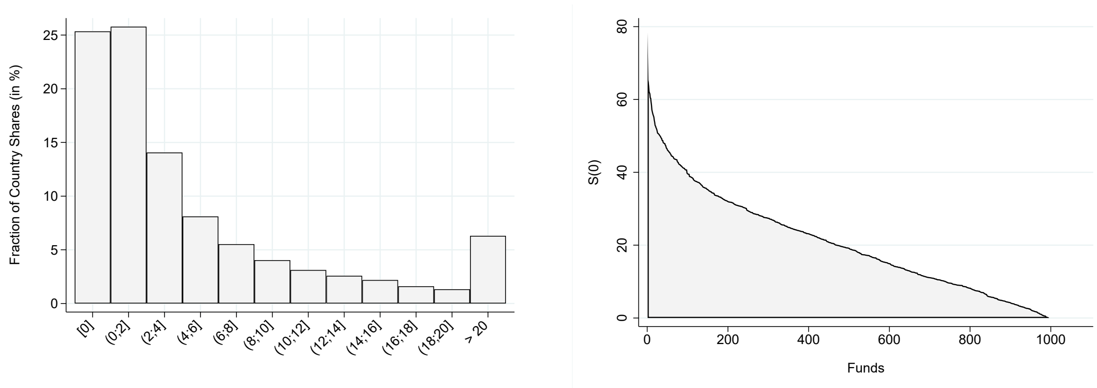
[1]Because funds do not report the country specific portfolio returns, I combine the data with dividend-adjusted equity returns compiled by MSCI.

[2]The EPFR data cover a large amount of mutual funds and is representative of all mutual funds [Jotikasthira et al., 2012]. Fratzscher [2012] writes “EPFR data [. . .] is the most comprehensive one of international capital flows, in particular at high frequencies and in terms of its geographic coverage at the fund level.”

[3]Argentina, Australia, Austria, Belgium, Brazil, Canada, Chile, China, Colombia, Czech Republic, Denmark, Egypt, Finland, France, Germany, Greece, Hong Kong, Hungary, India, Indonesia, Ireland, Israel, Italy, Japan, Korea South, Malaysia, Mexico, Netherlands, Norway, Peru, Philippines, Poland, Portugal, Russian Federation, Singapore, South Africa, Spain, Sweden, Switzerland, Taiwan, Thailand, Turkey, United Kingdom, United States.

fraction of zeroes at the fund level. Let the fraction of corner solutions at the fund level be $S(0)_i = 100 \cdot N0_i/(N_i T_i)$, where $N0_i$ counts the total number of corner solutions per fund. Figure 1b shows $S(0)_i$ in a descending order where the x-axis represents the funds. While some few funds have a high fraction of zero shares, approximately 89 funds - 8% of all funds - always invest positive shares.

Figure 1: CORNER SOLUTIONS



(a) DISTRIBUTION OF $z_{i,n,t}$

(b) $S(0)_i$ IN (%)

Notes: Figure 1a shows the total distribution of $z_{i,n,t}$. The last category represents all shares higher than 20%. The y-axis represents what percentage of the observations is in each category. Figure 1b shows $S(0)_i$ in a descending order where the x-axis represents the funds.

In the next section, I discuss the optimal portfolio shares derived from a model.

3 Portfolio Equations

I analyze the implication of those corner solutions through the portfolio equations given by solving a N-asset Markowitz mean-variance model with investor heterogeneity. Because of short-selling constraints the optimal share invested in country n, z_{int} ranges between 0 and 1 and $\sum_n z_{i,n,t} = 1$. I study two variations of this model. The first is a frictionless model, equation (4). The second portfolio regression adds frictions in the form of costly deviations from the past shares, equation (5). This portfolio friction is micro founded. The literature has found persistent effect of past shares in portfolio choice. The full models behind each equation are depicted in the appendix. The portfolio equations are

$$z_{i,n,t} = \max\{0, \bar{z}_{i,n} + b_{1,in} E_t er_{i,n,t,t+1}\} \tag{4}$$

$$z_{i,n,t} = \max\{0, \bar{z}_{i,n} + c_{1,in}(z_{i,n,t-1} - \bar{z}_{i,n}) + c_{2,in} E_t er^{\delta_{in}}_{i,n,t,t+K}\} \tag{5}$$

where $0 \leq z_{i,n,t} \leq 1$, $\sum_n z_{i,n,t} = 1$, $E_t er_{i,n,t,t+1}$ is the expected future equity return at horizon $t+1$ and $E_t er_{i,n,t,t+K}$ is the expected future equity return at horizon $t+K$. For any $s \geq 1$, the expected excess return is equal to $E_t er^{\delta_{in}}_{i,n,t,t+s} = E_t er^{\delta_{in}}_{n,t,t+s} - \sum_{m \neq n} z_{i,m,-n,t-1} E_t er^{\delta_{in}}_{m,t,t+s}$. This expression corresponds to how much an investor expects equity return to be in country n relative to the other countries in which he/she invests. I predict expected excess returns at horizon $t+s$ using macro and bond variables from t. Because portfolio demand at t - $z_{i,n,t}$ does not impact macro quantities and bond prices, the expected excess returns is exogenous. I use a 3-months interest rates, inflation rates, growth in dividends, earnings and book values and bond return. I detail the data and show the predictability regressions in the appendix. While in the theory $K = \infty$, in the empirical applications K is necessarily finite. I use as benchmark $K = 24$. The parameter $\delta_{in} = \beta * c_{1,in}$, where $\beta = 0.97$ is the time discount factor. While $\beta = 0.97$ may seem low with monthly data, the average turnover of portfolio managers is 2 percent per month [Kostovetsky and Warner, 2015]. An even lower β may need to be assumed if we take into account that many funds have short lives.

Corner solutions happen when negative expected excess returns push the optimal country share to zero. The higher the coefficients on expected excess returns, the more frequent are corner solutions. Because of the persistence of financial shocks, the investor may not invest for some periods. Corner solutions are also more likely to happen when the average share $\bar{z}_{in}$ and the past shares are small. Small weights are associated with funds which invest in a large amount of countries Finally, corner solutions are more likely to happen for funds with a low coefficient on past shares, equation (5). The higher the persistence, the more sticky are the portfolio shares.

The investor never invest in the countries which are not in his/her investment mandate. This obligation could be rationalize with costly entry. For instance, an investor may face very high entry cost when willing to invest in a country which is outside the investment scope. Hence, the investor is prevented from investing $n \notin N_i$.

Micro-Foundations

The coefficients b and c of equations (4) and (5) are linked to the underlying structural parameters of the model (see appendix for the derivation).

$$\widehat{\gamma}_i = \frac{1}{b_{1,in}\sigma^2_{in}} \tag{6}$$

$$\lambda_{in} = \frac{c_{1,in}}{c_{2,in}} \tag{7}$$

$$\gamma_i = \frac{1}{c_{2,in}\sigma^2_{in}}(1 - c_{1,in})(1 - \beta c_{1,in}) \tag{8}$$

where γ is risk aversion, σ^2 is the average variance of the excess return and λ represents the value of portfolio frictions. $\widehat{\gamma}$ is different than γ because of the portfolio frictions.

So all the parameters of the portfolio solution depend on both i and n. There are three sources of heterogeneity that make the parameters depend on i and n. The first is that investors vary in their risk-aversion. The second is that risk, about the variance of the excess return varies across i and n. The final is that the portfolio frictions λ_{in} varies across investors. The parameter $\bar{z}_{in}$, the mean portfolio, depends on risk, which varies across investors and countries. The weight on the lagged portfolios depends on risk-aversion, which varies across investors. The higher risk-aversion, the less weight they give to the past shares. It also depends on the variance of the excess return, which varies across investors and countries. Higher risk leads to a lower weight on the lagged portfolio. The coefficient on expected future excess returns, depends on risk-aversion (which varies across investors) and risk (which varies across investors and countries). Higher risk-aversion and higher risk leads to a weaker response to changes in expected excess returns. Finally, the discount factor δ_{in} depends on the same factors as the coefficient on past shares. Higher risk for example, which reduces the weight on lagged portfolios, also lowers the discount rate.

In the next section, I relate the occurrence of corner solutions with funds' heterogeneity. Specifically, funds which show different level of risk aversion, portfolio risk and have a lower value of portfolio friction.

4 Corner Solutions and Funds' Characteristics

As a result, corner solutions are more likely to occur for funds which (i) are more responsive to expected returns, (ii) invest in a large number of countries, $\bar{z}_{in}$ is low, and (iii) are less constrained by past shares. Bacchetta, Tièche, and van Wincoop [2020] show that active, small and broad funds feature those characteristics.

I explore the link between the corner solutions and funds' characteristics by first showing descriptive statistics. I split the sample in three ways: active versus passive funds, small versus large funds and broad versus specialized funds. I measure activeness with a measure of portfolio volatility defined as $V_i = \frac{100}{T_i} \sum_t \sum_n |z_{i,n,t} - z^{bh}_{i,n,t}|$. V_i sums the absolute changes from the buy-and-hold portfolio, which tracks how much and how frequent a fund actively rebalances its portfolio. Bacchetta, Tièche, and van Wincoop [2020] use the same formula. Fund size corresponds to the average assets under management (AUM_i). I measure how broad a fund is by the number of countries in which it invests. Table 1 shows the average fraction of zero country shares (in percent) by quarters of funds' characteristics. I split the funds by quarters for each category. For instance, Q1 (Q4) of the row *Activeness* represents the most passive (active) funds.

Table 1: FRACTION OF ZERO COUNTRY SHARES (IN PERCENT) BY FUNDS' CHARACTERISTICS

Quarters	Activeness	Broadness	Size
Q1	10.4	18.1	24.8
Q2	22.4	21.2	27.6
Q3	27.0	24.3	26.3
Q4	34.4	29.3	23.6

This Table shows the average fraction of zero country shares (in percent) by quarters of funds' characteristics. Q1 represents the bottom quarter while Q4 represents the top quarter. For instance, Q1 (Q4) of the row *Activeness* represents the most passive (active) funds.

More active and broader funds have a higher fraction of zero country shares, Table 1. However, the results of Table 1 are opaque as funds with a high fraction of zero country shares could be active, broad and small at the same time. I disentangle the effect of each funds' characteristics by regressing the fraction of corner solutions on those characteristics. I also add T_i as a regressor because funds which survive the longer are on average larger, more active and invest in more countries. I use the log of assets under management (AUM) because of very large funds. Equations (9) and (10) express the regression equations.

The dependent variable $S(0)_i$ takes the discrete value of 0 when a fund does always invest a strictly positive value in all countries N_i or a continuous value if the fund faces corner solutions (Figure 1b).

$$S(0)_i = \max\{0, S(0)_i^*\} \tag{9}$$

$$S(0)_i^* = a_0 + a_1 V_i + a_2 N_i + a_3 \ln(AUM_i) + a_4 T_i + \nu_i \tag{10}$$

I estimate the system of equations (9) and (10) with a Tobit model and compare the results to an OLS model. Because the distribution of $S(0)_i$ has a large mass of point at the discrete value of zero (as some funds always invest a strictly positive value in all countries N_i), OLS produce bias coefficients. Only a Tobit estimator can take into account all the partial effects and estimate consistent coefficients.

Table 2 shows the regressions results. Column (1) shows the Tobit estimates when T_i is not included. Column (2) includes T_i. There are 89 funds which always invest a strictly positive value in all countries N_i. Column (3) shows OLS on all values and column (4) shows OLS for strictly positive values. I label it OLS+. Robust standard errors are in parentheses.

Table 2: FUNDS' CHARACTERISTICS AND SHARE OF ZEROES

	(1) Tobit	(2) Tobit	(3) OLS	(4) OLS+
V_i	2.170***	1.886***	1.747***	1.536***
	(0.146)	(0.135)	(0.127)	(0.130)
N_i	0.251***	0.278***	0.247***	0.206***
	(0.052)	(0.048)	(0.044)	(0.048)
$\ln(AUM_i)$	0.653***	-0.250	-0.250	-0.325
	(0.247)	(0.243)	(0.224)	(0.237)
T_i		0.113***	0.105***	0.094***
		(0.010)	(0.010)	(0.010)
Observations	1084	1084	1084	995
R^2			0.323	0.262
Left-Censored Observations	89	89		

Robust standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.
Notes: Regressions with 1084 funds. Variables are the average between 2002:01 and 2016:7. The constant is included but not shown.

The fraction of corner solutions increases with portfolio volatility V_i and the investment universe N_i. However, fund size $\ln(AUM_i)$ is not significant, Column (2). Omitting T_i creates a bias which overestimates the impact of activity and estimates the impact of fund size with the wrong sign. Funds that live longer also have a higher fraction of zeroes. Both OLS estimates are close to the Tobit model as the fraction of corner solution is only 8%. Interestingly, OLS estimates are approximately equal to the Tobit estimates times (1-0.08), the fraction of non-corner observations.

In the next section, I describe which estimator is the best to estimate equations (4) and (5).

5 Estimate Portfolio Equations with Corner Solutions

Data on portfolio choice have many features which requires a careful examination. First, portfolio shares z_{int} are fractional: $0 \leq z_{int} \leq 1$. While in theory the portfolio share can be equal to one, in practice the shares are never equal to one thanks to international diversification. Second, the portfolio shares are log-normally distributed. Hence, OLS or Tobit estimators might be ill-suited. Third, portfolio weights are heteroskedastic. The variance of the portfolio shares given the explanatory variables is likely to be higher for larger shares. Finally, portfolio shares feature inertia. Hence, introducing a lagged dependent variable is important to capture slow adjustment in portfolio shares.

Taking those facts into account, the best estimator depends on whether there is a lagged dependent variable or not. When there is no lagged dependent variable, the best estimator is the Bernoulli quasi maximum likelihood estimator proposed by Papke and Wooldridge [1996] and then extended to panel data by Papke and Wooldridge [2008]. This estimator is consistent for linear and non-linear functional form between the dependent and the independent variables. It does not need a particular form. When the standard are corrected for heteroskedasticity, the inference is right. The Bernoulli QMLE is the best for fractional data. It can accommodate the log-normal distribution of the portfolio weights. It takes into account the heteroskedasticity without giving a particular form to the conditional variance.[4] However, this estimator is ill-suited when there is a lagged dependent variable. In that case, the best estimator is the two-limit random effect Tobit proposed by Loudermilk [2007]. She uses this estimator in an application to dividends payout. Papke and Wooldridge [2008] admit the Tobit estimation is adapted to dynamic applications in which we observe fractional data. I compare the results of the Bernoulli QMLE and the two-limit random effect Tobit to OLS: OLS on all shares (OLS) and OLS on the truncated sample of positive weights (OLS+).

Zeroes are also present in the international trade literature. It happens when one countries does not export a good with another country for instance. Since the seminal paper of Anderson and van Wincoop [2003], the international trade literature fit gravity equations after having log-transformed the data. This transformation drops the zeroes. Silva and Tenreyro [2006] develop a Poisson estimator to estimate those gravity equations. However, I cannot use Poisson regression as the conditional mean of portfolio shares is not given by an exponential

[4]The econometrician could give a function form to this heteroskedasticity. For instance, by using a logit or probit function form and deriving the conditional variance. Otherwise, the econometrican could correct the standard errors by using robust or by clustering the standard errors.

expression.

In the next section, I estimate the portfolio equations (4) and (5) using the econometric estimator discussed above.

6 Portfolio Regressions

I now estimate the portfolio equations

$$z_{i,n,t} = \max\{0, \bar{z}_{i,n} + b_{1,in} E_t er_{i,n,t,t+1}\} \quad (4)$$

$$z_{i,n,t} = \max\{0, \bar{z}_{i,n} + c_{1,in}(z_{i,n,t-1} - \bar{z}_{i,n}) + c_{2,in} E_t er^{\delta_{in}}_{i,n,t,t+K}\} \quad (5)$$

and link the estimated coefficients with the structural parameters driving portfolio decisions: the risk aversion, the variance of the excess returns and the cost of portfolio frictions. I estimate the portfolio regressions with the best-suited estimator and with OLS. I start by estimating equation (4). Then, I turn to estimating equation (5).

6.1 Frictionless Portfolio Equation

The frictionless portfolio equation is

$$z_{i,n,t} = \max\{0, \bar{z}_{i,n} + b_{1,in} E_t er_{i,n,t,t+1}\} \quad (4)$$

where the structural parameter $\widehat{\gamma}$ is equal to $\frac{1}{b_1 \sigma^2}$. The variance of the excess return σ^2 corresponds to the average variance of country returns given by MSCI. It is equal to 0.0053.

Table 3 report the results. Column (1) estimates the Bernoulli QMLE. Column (2) estimates OLS and column (3) estimates OLS on the strictly positive shares. I force the coefficient on $\bar{z}_{in}$ to be equal to one. In each regression, I include a constant. I cluster standard errors by month and domicile as Raddatz and Schmukler [2012].

Table 3: PORTFOLIO REGRESSIONS

	$z_{i,n,t} = \max\{0, \bar{z}_{i,n} + b_{1,in} E_t er_{i,n,t,t+1}\}$		
	Bernoulli QMLE (1)	OLS (2)	OLS+ (3)
$er_{i,n,t,t+1}$	6.567*** (0.310)	0.251*** (0.035)	0.363*** (0.053)
$\widehat{\gamma}$	28.7	751.7	519.8
Observations	976,736	976,736	734,890

Clustered standard errors by month and domicile in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Regressions for 44 countries over the interval 2002:01-2016:07. A constant is included but not shown.

The Bernoulli QMLE estimator reports a coefficient of 6.567 which gives a risk aversion of 28.7. The other ill-suited estimators underestimate the coefficient on expected excess returns with a factor of 18 to 26. Those estimators overestimate the risk aversion. Micro studies and experiments find coefficients of risk aversion ranging from 3 to 10. Even though, the risk aversion estimated by the Bernoulli QMLE is still too high it is closer to the micro estimates. The next portfolio equation introduces a portfolio friction. Bacchetta, Tièche, and van Wincoop [2020] find that introducing portfolio frictions help getting risk aversion closer to the micro estimates.

6.2 Portfolio Friction

The portfolio equation with portfolio friction is

$$z_{i,n,t} = \max\{0, \bar{z}_{i,n} + c_{1,in}(z_{i,n,t-1} - \bar{z}_{i,n}) + c_{2,in} E_t er^{\delta_{in}}_{i,n,t,t+K}\} \tag{5}$$

where the structural parameter γ is equal to $\frac{1}{c_2 \sigma^2}(1 - c_1)(1 - \beta c_1)$ and the value of portfolio friction λ is equal to $\frac{c_1}{c_2}$.

Table 4 report the results. Column (1) estimates the random effect Tobit. Column (2) estimates OLS and column (3) estimates OLS on the strictly positive shares. As in Table 3, I force the coefficient on $\bar{z}_{in}$ to be equal to one. In each regression, I include a constant. I cluster standard errors by month and domicile as Raddatz and Schmukler [2012].

Table 4: PORTFOLIO REGRESSIONS

$z_{i,n,t} = \max\{0, \bar{z}_{i,n} + c_{1,in}(z_{i,n,t-1} - \bar{z}_{i,n}) + c_{2,in}E_t er^{\delta_{in}}_{i,n,t,t+K}\}$			
	Tobit	OLS	OLS+
	(1)	(2)	(3)
$z_{i,n,t-1}$	0.901***	0.925***	0.920***
	(0.001)	(0.003)	(0.003)
$er^{\delta}_{i,n,t,t+24}$	0.188***	0.071***	0.119***
	(0.019)	(0.015)	(0.029)
γ	11.4	20.5	13.7
λ	4.8	13.0	7.7
Observations	976,736	976,736	734,890

Clustered standard errors by month and domicile in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Regressions for 44 countries over the interval 2002:01-2016:07. A constant is included but not shown.

The Tobit reports a coefficient of 0.188 for expected excess returns and a persistence of 0.901. Those results give a risk aversion of 11.4 and a portfolio friction of 4.8. Although it is still higher than micro estimates, it is closer than OLS results. Both OLS overestimate the persistence of past shares and hence underestimate the coefficient on the expected excess return. Therefore, OLS overestimate both the parameter of portfolio friction and risk aversion.

7 Discussion of Results

Estimating portfolio equations requires a careful examination of the data. When the econometric model is correctly specified, the structural parameters governing the theoretical choice of portfolio shares are closely estimated to their micro estimates. In this application to international portfolio choice, introducing portfolio frictions brings the structural parameters closer to their micro estimates. The large sample corrects some of the bias introduced when estimating the portfolio equation with an ill-suited estimator. However, as some bias remain, it is still important to take into account the corner solutions and apply the right estimator.

In ongoing revisions, I estimate the distribution of the structural parameters. I also separate the analysis by types of funds.

8 Conclusion

The objective of the paper is to study the presence and the implication of corner solutions in individual data on international portfolio choice. After having identified the investor investment set, I show that corner solutions represent 25% of the data. Given the structure of the data and the economic regression, the best estimator is the Bernoulli quasi maximum likelihood when there is no lagged dependent variable and the Tobit random effect when there is a lagged dependent variable. In the portfolio equations, the linear estimators produce coefficients with a large bias. Hence, taking corner solutions into account matter for the econometric analyzes of portfolio shares.

Both the discussion on how to find corner solutions and the econometric estimator can help researchers in several fields, including non-economic fields. However, a limitation applies to problems in which the number of alternatives is infinite. For instance, decisions over an infinite number of assets.

A Model of Portfolio Choice

The model of portfolio choice is available under request.

B Predictability of Excess Returns

The regressions of excess return predictability are available under request.

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